

BALANCING

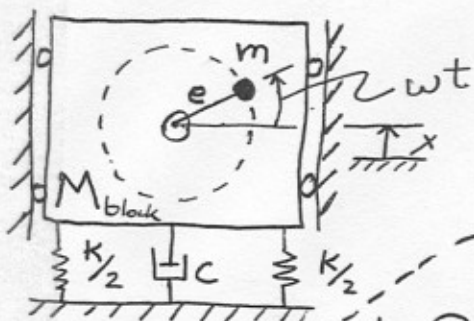
18-1

- Rotating parts should be designed to be inherently balanced by their geometry. An unbalance in machines is attributed to irregularities such as machining errors, size variation in bolts, nuts, welds, etc., wear, and particle accumulation.
- To balance a shaft, disk, or gear, we need to determine the size and location of the eccentric mass. Then, to correct the problem we can _____.

Unbalance in rotating machines occurs when a rotating member does not possess symmetry wrt. mass

Examples: _____

Consider the system with a rotating imbalance of mass, m

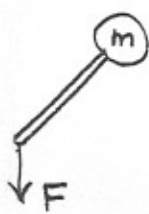


The rotation of the eccentric mass, m exerts a harmonic force on the block, M_{block} (Assuming $\omega = \text{constant}$)

Recall, for a mass on a rod rotating about a fixed point, the only force acting on the mass is the tension in the rod. Because it has circular motion the acceleration

of the mass is $\vec{a} = \underline{\hspace{2cm}}$ but $\vec{v} = \omega R$ so, $\vec{a} = \underline{\hspace{2cm}}$
and the centripetal force is given by $F_{\text{rod}} = \underline{\hspace{2cm}}$

Now let's draw the FBD for m and M_{block} assuming $x > 0, \dot{x} > 0$



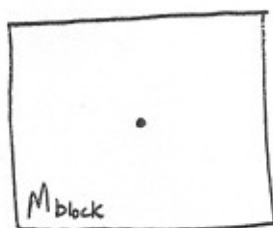
The position of the imbalance, m is given by $z(t)$

$$z(t) = x(t) + e \sin \omega t$$

$$\sum F_x = -F = m \ddot{z}(t) = m (\ddot{x}(t) - e \omega^2 \sin \omega t)$$

$$\text{so, } F = -m \ddot{x} + m e \omega^2 \sin \omega t$$

FBD M_{block}



Applying Newton's Law to M_{block}

$$\sum F_x = M_{\text{block}} \ddot{x} = -kx - c\dot{x} + F$$

$$M_{\text{block}} \ddot{x} + c\dot{x} + kx = -m \ddot{x} + m e \omega^2 \sin \omega t$$

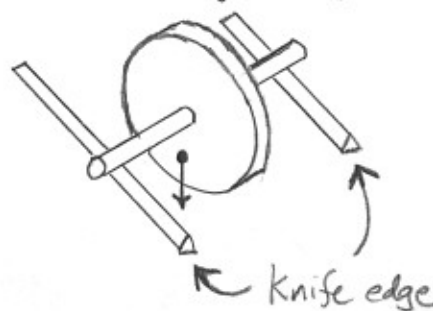
$$\text{Let } M = \text{total system mass} = M_{\text{block}} + m$$

$$\text{Then, } M \ddot{x} + c\dot{x} + kx = (m e \omega^2) \sin \omega t$$

The term $(m e \omega^2)$ is equivalent to the constant amplitude term F_0 that we typically see for SDOF systems, except now the force amplitude depends on frequency.

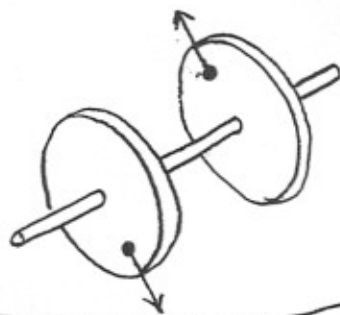
Static Balance (single-plane balance)

The masses which are generating the inertia forces are in (or nearly in) the same plane. Static balancing can be applied to a single gear, pulley, bicycle wheel or a thin flywheel. Typically the imbalance can be detected by using a static test. The wheel will roll to a position where the heavy point is directly below the axle.



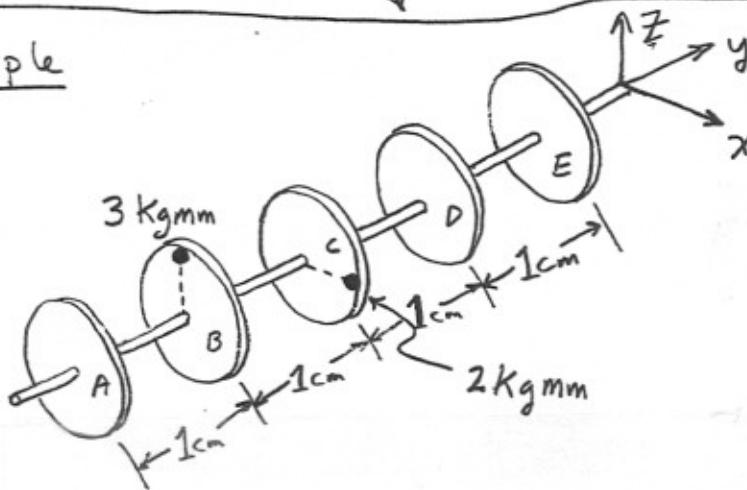
- For a car tire and rim balancing is necessary from time to time. A static wheel balance entails placing the wheel in a horizontal plane suspended by a cone through its center hole. A bubble level is attached to the wheel and weights are added to the rim until the wheel sits level. This type of balancing is generally no longer done because it neglects the effect of unbalanced moments.

- Dynamic Unbalance - When the unbalanced masses appear in more than one plane, the resultant is a force and rocking moment. The problem will not be remedied by a static test. A static test won't detect the rocking moment.



← statically balanced but not dynamically balanced.

Example

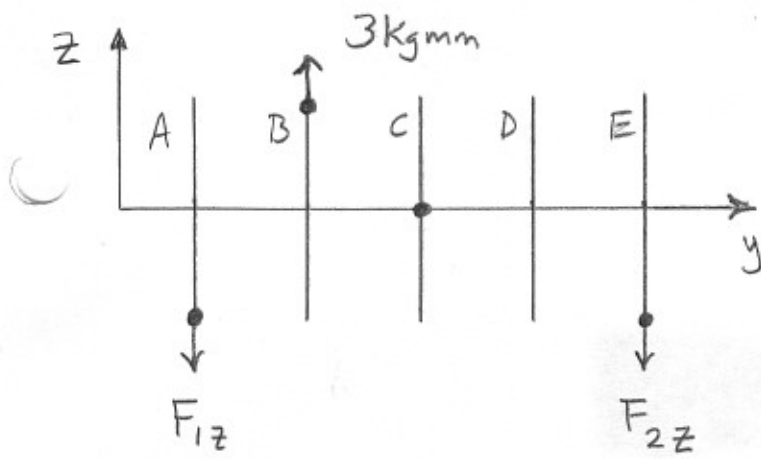


Balance the shaft by placing masses on disk A and E.

Solution

First we will balance disk B and then disk C

(Side View)



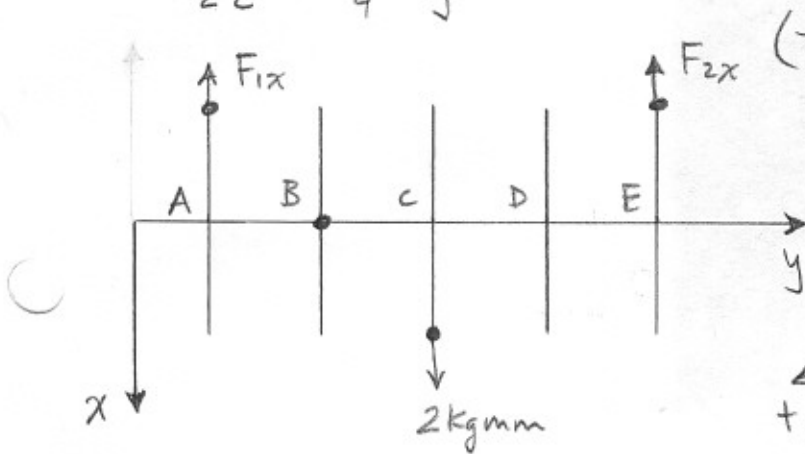
$$\sum F_z = 3 - F_{1z} - F_{2z} = 0$$

$$F_{1z} + F_{2z} = 3 \quad (1)$$

$$\uparrow \sum M_B = F_{1z}(1) - F_{2z}(3) = 0$$

$$F_{1z} = 3F_{2z} \quad (2) \quad \text{Combining (1+2)} \rightarrow F_{1z} = \frac{9}{4} \text{ Kgmm}$$

$$F_{2z} = \frac{3}{4} \text{ Kgmm} \quad \text{Now solve the forces to balance disk C}$$



(Top View)

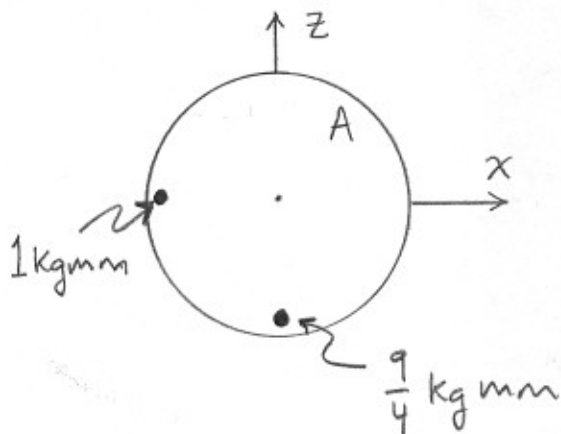
$$\sum F_x = 2 - F_{1x} - F_{2x} = 0$$

$$F_{1x} + F_{2x} = 2$$

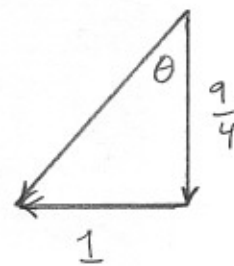
$$\leftarrow \sum M_C = -F_{1x}(2) + F_{2x}(2) = 0$$

$$F_{1x} = F_{2x} = 1 \text{ Kgmm}$$

But if we only wanted to add a single weight to disk A



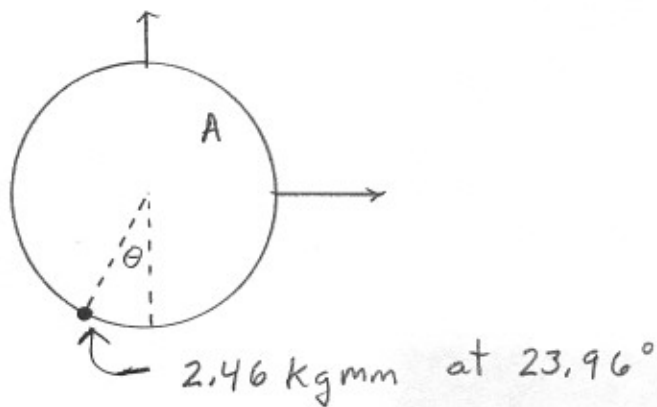
$$\sqrt{1^2 + \left(\frac{9}{4}\right)^2} = 2.46 \text{ Kgmm}$$



$$\theta = \tan^{-1}\left(\frac{1}{9/4}\right)$$

$$\theta = 23.96^\circ$$

Instead of using two additional weights we can use one



To perform a dynamic balance of an automobile wheel the following set up is typically used.

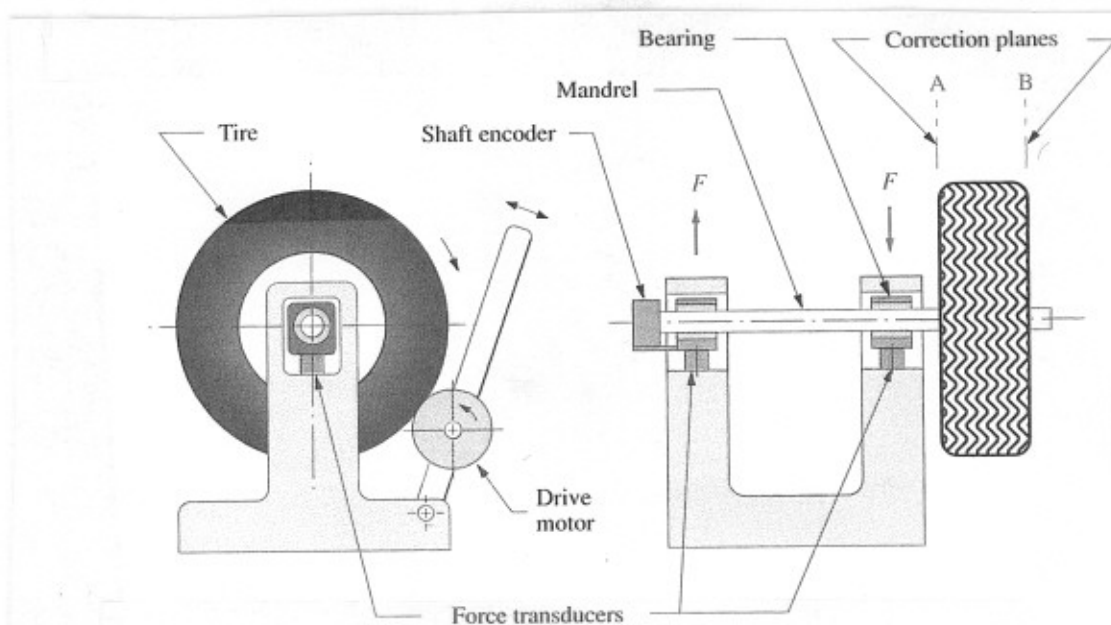


FIGURE 12-12
A dynamic wheel balancer

The wheel is driven up to speed by a motor and then allowed to spin freely. Two force transducers and a shaft encoder measure the forces and wheel speed and position. Based on the equations $\sum F = 0$ and $\sum M = 0$, the additional masses and orientation of the balancing weights are determined. The correction radius is that of wheel rim.